APPENDIX

Davis[3] has shown that the neutral particle $\bar{\nu}_\beta$ that appears in negatron emission is different from the neutral particle ν_β that appears in positron emission. This difference was established, using antineutrinos from a reactor, by demonstrating that the cross section for $Cl^{37}(\bar{\nu}_\beta,e^-)Ar^{37}$ was much less than would be expected, on the basis of the measured Ar^{37} electron-capture lifetime, if $\bar{\nu}_\beta$ were equal to ν_β.

Previous estimates of the expected cross sections for $Cl^{37}(\bar{\nu}_\beta,e^-)Ar^{37}$ have not included the possibility of excited-state transitions; such transitions may be included using formulas (29) and Table IV of the present work. For use in this connection, we present in Table VII values of $w_e^2G(18,w_e)$ for neutrino energies that are relevant to Davis's reactor experiment.

TABLE VII. Values of $w_e^2G(18,w_e)$.

Q_ν(MeV)[a]	$w_e^2G(18,w_e)$	Q_ν(MeV)[a]	$w_e^2G(18,w_e)$
0.814	0.0	3.000	52.8
0.850	1.4	4.000	99.2
0.875	1.7	5.000	160
0.900	1.9	6.000	235
0.950	2.4	7.000	324
1.000	2.9	8.000	426
1.500	10.3	10.000	675
2.000	20.7	12.000	1004
2.500	35.8	14.000	1539

[a] Q_ν(MeV) $=0.303+W_e$(MeV).

PHYSICAL REVIEW VOLUME 135, NUMBER 1B 13 JULY 1964

"Old" Resonance Model for 3π Decays of K and η

A. N. MITRA AND SHUBHA RAY
Department of Physics and Astrophysics, University of Delhi, Delhi, India

(Received 6 February 1964; revised manuscript received 4 April 1964)

In view of the current experimental interest in the three-pion modes of K and η decays, a phenomenological model of a π–π resonance in $I=0$, proposed several years ago by one of us, is re-examined in the context of the present experimental data. It is found that the model satisfactorily explains all the available features of τ and τ' decays if the resonance occurs at an energy around 400 MeV with a width 85–90 MeV. The corresponding parameters with a somewhat smaller width ($\sim$75–80 MeV) also give satisfactory agreement with the experimental results for η decay. This model corresponds to a *repulsive* $I=0$ π–π interaction at low energies and is therefore not in disagreement with the earlier dispersion theoretical predictions for τ decay, using final-state interactions. It also has a strong resemblance to the σ model of Brown and Singer.

1. INTRODUCTION

THE old problem of τ decay has acquired renewed interest in recent times through the observation of η-decay events and their great similarity with the former. The current theoretical picture has been aptly summarized by Kacser,[1] namely, there are two broad alternatives (i) the pion-pole model[2] in which the momentum dependence of the weak-interaction structure makes itself felt through derivative couplings (for p-wave resonances) and (ii) the final-state interaction (fsi) model in which the weak interaction has no structure (i.e., has no momentum dependence). The fsi model got into disrepute after the dispersion theoretic calculations of Khuri and Treiman[3] showed that the calculated energy spectrum of the unlike pion was incompatible with the requirement of a stronger $I=0$ $\pi-\pi$ force than $I=2$, as believed from other evidence (e.g., low-energy $\pi-N$ scattering). However, the fsi model seems to have recovered quite a bit of lost ground through the suggestion of a $J=I=0$ $\pi-\pi$ resonance (called σ) by Brown and Singer[4] who have shown that such a "particle" can satisfactorily explain the energy spectrum in the τ and η events. The recent experimental verification by Crawford *et al.*[5] of the π^0 spectrum in $\eta\rightarrow\pi^++\pi^-+\pi^0$ in terms of the Brown-Singer hypothesis provides quite impressive support in its favor, though, of course, it does not confirm the existence of σ.

As the fsi model is now doing rather well (at least for the time being), we would like to draw attention to a phenomenological model proposed in connection with τ decay by one of us (ANM) several years ago[6] when the data were rather poor. Subsequently, a refined version of this model[7] which we shall refer to in this note as "resonance model," was reported by the same author (ANM)[8] to produce a qualitatively correct

[1] C. Kacser, Phys. Rev. **130**, 355 (1963).
[2] M. A. Baqi Bég and P. de Celles, Phys. Rev. Letters **8**, 46 (1962). See also G. Barton and S. P. Rosen, Phys. Rev. Letters **8**, 414 (1962); Riazuddin and Fayyazuddin, *ibid.* **7**, 464 (1961).
[3] N. Khuri and S. Treiman, Phys. Rev. **119**, 1115 (1960). Also, R. Sawyer and K. C. Wali, *ibid.* **119**, 1429 (1960).
[4] L. Brown and P. Singer, Phys. Rev. Letters **8**, 460 (1962).
[5] F. S. Crawford, Jr., R. S. Grossman, L. J. Lloyd, L. R. Price, and E. C. Fowler, Phys. Rev. Letters **11**, 564 (1963).
[6] A. N. Mitra, Nucl. Phys. **6**, 404 (1958); referred to as A.
[7] A. N. Mitra, Nucl. Phys. **18**, 502 (1960); referred to as B.
[8] A. N. Mitra, in *Proceedings of the 1960 International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960).

trend for the unlike-pion energy spectrum in τ decay provided the "resonance" was in the state $I=0$ and the resonance energy was substantially higher than the maximum energy allowed for each pion. This model, unfortunately, did not receive much attention at that time, presumably because its prediction was just the opposite of the dispersion theoretical results reported at the same time. However, now that there seems to be a good deal of experimental interest in the Brown-Singer model, we feel that it may be in order to put on record the predictions of this "old" model in the context of τ- and η-decay results.

The formalism describing the resonance model for interaction is completely contained in Refs. 6 and 7 which we refer to as A and B, respectively. However, as it appears from the current literature that these two papers did not receive much attention in the context of τ-decay results, it may be useful here to summarize the main results, viz., (1) the energy and angular distributions of the unlike pion, and (2) the branching ratios for certain pionic modes, for both the K-meson and η-meson cases. In Sec. 2, we give a brief account of the model and also collect the various formulas relevant to the above cases, on the basis of results derived in A and B. In Sec. 3, the results of this model are compared with experimental data. Section 4 summarizes the main conclusions and also attempts to reconcile this model with the results of earlier dispersion theoretic and final-state interaction calculations.

2. NECESSARY FORMALISM

The analysis in A and B, of the three-pion decay modes of k, follows the scheme of Dalitz,[9] according to which the pions are distinguished by the magnitudes of their momenta rather than their charges. This makes it possible to take account of the important effect of Bose statistics for the τ and τ' modes in a particularly convenient manner. The Dalitz scheme is of course equivalent to the ordinary description in which the pions are distinguished by their charges, for the cases $\tau^0\to\pi^++\pi^0+\pi^-$ or $\eta\to\pi^++\pi^-+\pi^0$, where the decay particles are all distinct.

As in A, let $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ be momenta of the three pions in *descending* order of magnitude and the isospins associated with $\mathbf{p}_2$ and $\mathbf{p}_3$ be combined to give a resultant isospin I (equal to 0, 1, or 2). This composite is now combined with the remaining pion of momentum $\mathbf{p}_1$ to produce a total isospin T for the 3π system, described by the various functions $\chi_I{}^T$ ($I=0$, 1, 2 and $T=1$, 2, 3). The most general amplitude for the process is then expressible in the notation of A as:

$$A=\tfrac{1}{6}\textstyle\sum_P(F_0\chi_0{}^1+F_1\chi_1{}^1+F_2\chi_2{}^1 +G_1\chi_1{}^2+G_2\chi_2{}^2+H_2\chi_2{}^3)\,, \quad (1)$$

where F, G, H are all functions of $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, and $\sum_P$ refers to all possible permutations of these arguments.

[9] R. H. Dalitz, Proc. Phys. Soc. (London) **A69**, 527 (1956).

If one assumes the $\Delta T=\frac{1}{2}$ rule strictly, only the functions F_I survive. We are here interested in considering a model in which only the $I=0$ amplitudes for two pions play the dominant role in producing the various decay characteristics so that in this case the $\Delta T=\frac{1}{2}$ rule is automatically satisfied, but in addition the functions F_1 and F_2 drop out of (1). As for the function F_0, the following resonance structure" was assumed in B:

$$W_1\equiv F_0(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)=\tfrac{1}{2}\gamma[\omega_r-\omega_{23}-\tfrac{1}{2}i\gamma k_{23}]^{-1}\,, \quad (2)$$

where $2\omega_r=2(r^2+\mu^2)^{1/2}$ is the total resonance energy and $(\pm\mathbf{k}_{23},\,\omega_{23})$ are the 4-momenta of the pions $\mathbf{p}_2$ and $\mathbf{p}_3$ in their own c.m. frame; γ is a dimensionless parameter related to the full width Γ_r at resonance by

$$\Gamma_r=2r\gamma=2\gamma(\omega_r{}^2-\mu^2)^{1/2}. \quad (3)$$

Similar expressions W_2 and W_3 hold when the "resonances" are considered as between the pairs $(\mathbf{p}_3,\mathbf{p}_1)$ and $(\mathbf{p}_1,\mathbf{p}_2)$, respectively. It may be noted that the functions W_i incorporate the correct threshold behavior of the width parameter Γ (as a function of energy), and this happens to be rather important for the fits to the experimental data. To list some of the important kinematics involved, we have

$$\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3=0\,,\quad \omega_1+\omega_2+\omega_3=M\,, \quad (4)$$

$$4\omega_{23}{}^2=4\mu^2+4k_{23}{}^2=M^2-2Mt_1-2M\mu+\mu^2\,, \quad (5)$$

$$\equiv 4\mu^2+2M(t_m-t_1)\,, \quad (6)$$

where

$$t_i=\omega_i-\mu\,, \quad (7)$$

$$2Mt_m=(M-3\mu)(M+\mu)\,. \quad (8)$$

t_m is the maximum kinetic energy available to each pion. The total amplitude A of Eq. (1) is now expressible as

$$A=\tfrac{1}{3}[W_1\chi_0{}^1(1;23)+W_2\chi_0{}^1(2;31)+W_3\chi_0{}^1(3;12)]\,, \quad (9)$$

where

$$\chi_0{}^1(i;jk)=3^{-1/2}u_i(u_jw_k+u_kw_j-v_jv_k)\,, \quad (i,j,k=1,2,3)\,, \quad (10)$$

and u, v, w are the pion isospin functions for charges $+1$, 0, -1, respectively. From (9), the amplitudes A_1, A_2, A_3 in which the unlike pion in τ decay has the respective momenta $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ are given by[10]

$$A_i=\tfrac{1}{3}3^{-1/2}(W_j+W_k)\,,\quad (i,j,k=1,2,3)\,. \quad (11)$$

Similarly for τ' decay, the corresponding amplitudes A_1', A_2', A_3' in which the unlike pion has the respective

[10] These are slightly different from the corresponding expressions for A_i given in A or B where the contributions from the [2,1] amplitudes which are *odd* with respect to the interchange of $\mathbf{p}_2$ and $\mathbf{p}_3$ were neglected. These terms, being proportional to (w_2-w_3) etc., would contribute small (p-wave) effects, and hence would not bring about any important numerical modifications in the results derived in those papers. However, we have now rectified this omission, and the corrected expressions look simpler in structure.

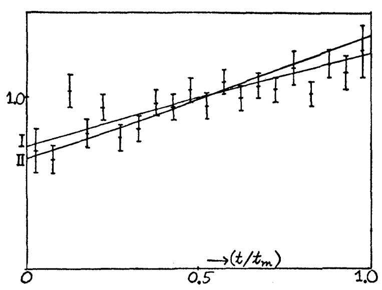

FIG. 1. Reduced energy spectrum of the unlike pion in τ decay. The experimental points are those of Smith *et al.* (Ref. 11). The theoretical curves I and II correspond to $\gamma^2=0.25$, 0.1, and $2\omega_r=397$ MeV.

momenta $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ are deducible from (9) as:

$$A_i'=-\tfrac{1}{3}3^{-1/2}W_i. \tag{12}$$

Now the energy distribution of the unlike pion in $k_{\pi 3}$ decay is given by the formula (see Appendix I of B)

$$\sigma(t),\ \sigma'(t)=\sum_{i=1}^{3}\int\{|A_i|^2,\ |A_i'|^2\}\delta(t+\mu-\omega_i)d\tau \tag{13}$$

where $d\tau$ is the phase-space volume element and the expressions (11) or (12) are substituted in (13) for τ or τ', respectively. Evaluation of the integrals can be performed by the methods outlined in the Appendices of A and B and the unnormalized functions are

$$\sigma(t)=\eta(\chi_m{}^+)-\eta(\chi_m{}^-), \tag{14}$$

$$\sigma'(t)=\xi(t)R_m, \tag{15}$$

where

$$R_m=Q^{-1}(\omega^2-\mu^2)^{1/2}(Q^2-4\mu^2)^{1/2}, \tag{16}$$

$$Q^2=4\mu^2+2M(t_m-t), \tag{17}$$

$$\begin{aligned}\xi(t)&=|W|^2\\&=\gamma^2[(Q-2\omega_r)^2+\tfrac{1}{4}\gamma^2(Q^2-4\mu^2)]^{-1},\end{aligned} \tag{18}$$

$$\eta(\chi_m{}^\pm)=(4\mu/M)\gamma(r^2-\tfrac{1}{4}\gamma^2\mu^2)^{-1/2}\tan^{-1}\chi_m{}^\pm, \tag{19}$$

and

$$\chi_m{}^\pm=\tfrac{1}{4}(\gamma\mu)^{-1}(r^2-\tfrac{1}{4}\gamma^2\mu^2)^{-1/2}\times[M\omega-3\mu^2-4r^2+2\mu^2\gamma^2\pm MR_m].$$

The quantity R_m represents just the phase-space distribution for the pion energy, so that the quantities of direct experimental interest are the "reduced spectra" $\sigma(t)/R_m$ and $\sigma'(t)/R_m$, apart from a normalization.

For the angular distribution of the unlike pion in τ decay let $\phi=\cos^{-1}\lambda$ be the angle between the π^- in the rest frame of τ^+ and the relative momentum vector for the two π^+'s. Then according to the interpretations of the vectors $\mathbf{p}_i$, the required distribution (unnormalized) is given by

$$\Phi(\lambda)=\sum_{i=1}^{3}\int d\tau|A_i|^2\delta(\lambda-\hat{p}_i\cdot\hat{R}_i) \tag{20}$$

where

$$\mathbf{R}_i=\mathbf{p}_j-\mathbf{p}_k.$$

This quantity, which can be evaluated by the method of Appendix II of B, works out as

$$\Phi(\lambda)=I_+(t_m)+I_-(t_m), \tag{21}$$

where

$$I_\pm(t)=\int_0^t dtR_m\gamma^2\{x_\pm{}^2+\gamma^2(\mu x_\pm+r^2)\}^{-1}$$

and

$$4\mu x_\pm=Mt+M\mu-3\mu^2-4r^2\pm M\lambda R_m. \tag{22}$$

The energy distribution $\sigma_0(t)$ of the neutral pion in $\tau^0\to\pi^++\pi^0+\pi^-$ or $\eta\to\pi^++\pi^0+\pi^-$ is given by

$$\sigma_0(t)=\xi(t)R_m\quad(0\leqslant t\leqslant t_m), \tag{23}$$

which is the same as Eq. (15) for $\sigma'(t)$, except for the modification $M_k\to M_\eta$ in the formulas for R_m and $\xi(t)$ in the case of η. This is, of course, expected since an $I=0$ resonance can occur only in the pair $(\pi^0-\pi^0)$ for τ' and $(\pi^+-\pi^-)$ for τ^0 or η. Finally, for completeness the branching ratios for the τ' and τ modes or η' and η modes $(\eta'\to3\pi^0)$, can be worked out, following the method of Appendix I of A, and the results are

$$R(\tau'/\tau)=\tfrac{1}{2}I'/(I+J), \tag{24}$$

where

$$I=\int|W_i|^2d\tau,\quad J=\mathrm{Re}\int W_j{}^*W_kd\tau \tag{25}$$

and I' is the same integral as I, except for the replacement of the charged pion mass by that of the neutral one in the resonance functions and the modification $t_m\to t_m'$ to take account of the larger Q value. Similarly

$$R(\eta'/\eta)=1.1[\tfrac{1}{2}+(J_\eta/I_\eta)], \tag{26}$$

where I_η and J_η are again given by (25) with $M_\tau\to M_\eta$, and the phase-space correction which is less important in this case because of the higher Q value, has been incorporated by the *ad hoc* factor 1.1.[5]

3. COMPARISON WITH EXPERIMENT

For comparison with the available experimental data, we have considered a variation of the width parameter γ in the region $0.08\leqslant\gamma^2\leqslant0.25$ corresponding to a resonance width 78–125 MeV, and that of the resonance energy $2\omega_r$ around the value $2\sqrt{2}\mu$ (≈397 MeV).

The reduced energy distribution $\sigma(t)/R_m$ of the unlike pion in τ decay is shown in Fig. 1, along with the experimental points of Smith *et al.*,[11] both normalized to unit total area.[12] At $2\omega_r=397$ MeV, the curve with the higher γ^2 is too flat for the data, but the one with $\gamma^2=0.1$ ($\Gamma_r\approx87$ MeV) seems to give a good fit to the

[11] L. T. Smith *et al.*, Phys. Letters **2**, 204 (1962); these data appear to be more exhaustive than the earlier ones whose references would be found here.

[12] Very recently it has come to our notice that T. Huetter, E. L. Koller, S. Taylor, P. Stamer, and J. Grauman, Bull. Am. Phys. Soc. **9**, 23 (1964), have analyzed 1000 τ events for the energy spectrum of π^+, but the details are not available to us at the time of writing this paper.

experimental points. Lowering of γ^2 much below this value would, however, start bringing strong deviation from linear behavior, in disagreement with experiment. A variation of $2\omega_r$ by about 30 MeV on either side of $2\sqrt{2}\mu$ (curves not shown) has not been found to cause any significant change in the shape of the spectrum. This is of course expected since the available phase-space energy is far below the resonance energy in this case.

The reduced energy distribution $\sigma'(t)/R_m$ of the charged pion in τ' decay is shown in Fig. 2, along with the experimental points of Giacomelli *et al.*,[13] both normalized to unit area. These data, based on 219 τ' events, are about the most exhaustive ones available to the authors at the moment[14] and they seem to represent an improvement over the statistics.[15] One feature which distinguishes the τ' data from the τ data, in spite of poorer statistics in the former, is that the unlike pion spectrum from τ' shows a sharper variation with energy (with a decreasing trend) than the unlike pion spectrum from τ (Fig. 1). In terms of our model, we can understand the sharper variation with energy in the case of the τ' spectrum, as follows: The δ functions appearing in (13) fully preserve the resonance structure $|W_i|^2=\xi(t)$ in the case of τ' where $A_i'\alpha W_i$. However, for τ, where $A_i\alpha(W_j+W_k)$, the δ-function integrations tend to *smear out* the resonance structures. In this respect it appears that the τ' spectrum could provide a more sensitive test of the model than the τ spectrum. The curve with $\gamma^2=0.10$ gives a fairly sharp drop with energy and thus appears to fit the data somewhat better than the one with $\gamma^2=0.25$. A further decrease in γ^2, however, brings about too sharp a drop near $t=0$, in disagreement with the experimental trend. A variation of the resonance energy by about 30 MeV has again not been found to produce any significant shift in the theoretical curves. These curves also represent the reduced spectrum of the π^0 in τ^0 decay, but the available experimental points[16] are not shown.

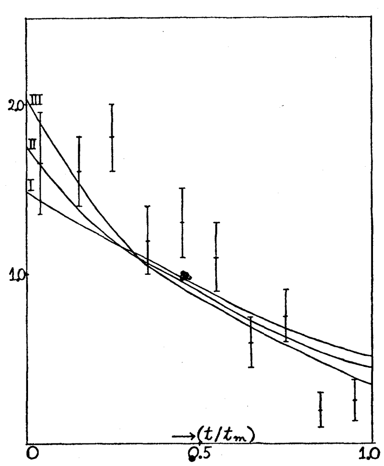

FIG. 2. Reduced energy spectrum of the charged pion in τ' decay, versus the experimental points of Giacomelli *et al.* (Ref. 13). The theoretical curves I, II, III are for $2\omega_r=397$ MeV and $\gamma^2=0.25$, 0.1, 0.05.

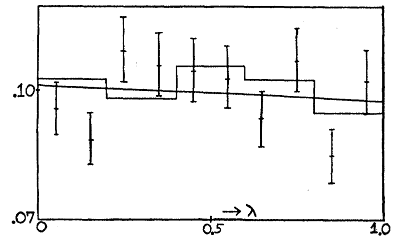

FIG. 3. Angular distribution of the unlike pion in τ decay, for different values of γ^2. The experimental points are those of Smith *et al.* (Ref. 11). The histogram represents the data of McKenna and O'Connell (Ref. 17).

Figure 3 shows the angular distribution of the unlike pion in τ decay, according to the formula (21). The calculated curves are surprisingly flat and show almost negligible variation with the width, the spectrum decreasing by about 2% between $\lambda=0$ and $\lambda=1$. The experimental points of Smith *et al.*[11] seem to indicate a stronger decrease with λ, at least in the middle region, but there are many scattered points near $\lambda=0$ and 1. While we are unable to comment on the reliability of these data, we feel very strongly, however, that the angular distribution should be much less sensitive to the resonance structure than the energy distributions, for the same reason as pointed out in the earlier paragraph. Indeed, an inspection of Eq. (20) would show that the "smearing out" of the resonance structures is now brought about through the angular δ functions which interfere more strongly with the functions W_i, than do the energy δ functions. For comparison we have also reproduced in Fig. 3 the histogram data of McKenna *et al.*[17] as given in Fig. 1 of B. These data, which are extremely flat, seem to be in conformity with our conclusions. More detailed experimental analysis of this question would be clearly welcome, in so far as it would provide a rather specific test of this prediction which seems to be a characteristic of the s-wave resonance model.[18]

The reduced energy distribution of the neutral pion in η decay, which is again given by the function $\xi(t)$ of Eq. (18) (with $M_\tau \to M_\eta$) is shown in Fig. 4, along with the corresponding "reduced" data of Crawford *et al.*[5] based on 97 events. This sample is claimed to be a relatively pure one, compared with, e.g., the data of Berley *et al.*,[19] though its statistics are rather poor. The data seem to be fairly well fitted with $\gamma^2=0.1$ and

[13] G. Giacomelli *et al.*, Phys. Letters **3**, 346 (1963).

[14] It has just come to our notice that the Berkeley-Wisconsin group [G. E. Kalmus, A. Kernan, and W. M. Powell, Bull. Am. Phys. Soc. **9**, 34 (1964)] have analyzed 2000 τ' events, but these are not immediately available to us.

[15] See, e.g., J. K. Bøggild *et al.*, Nuovo Cimento **19**, 621 (1961).

[16] D. Luers, I. S. Mittra, W. J. Willis, and S. S. Yamamoto, Phys. Rev. Letters **7**, 255, 361 (1961).

[17] S. McKenna and O'Connell [private communication (Ref. 7), May 1960].

[18] A p-wave model (Ref. 2) could in principle predict a stronger variation with λ, since in this case, additional contributions to λ-dependent terms would come from the *numerators* of the resonance functions.

[19] D. Berley, D. Colley, and J. Schultz, Phys. Rev. Letters **10**, 114 (1963).

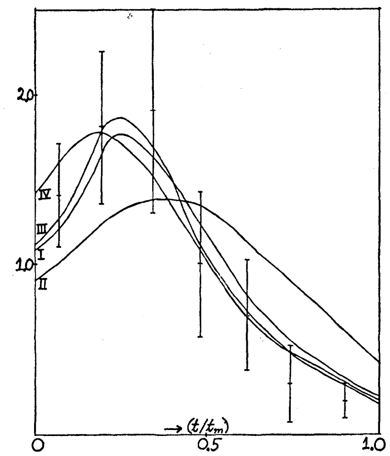

FIG. 4. Reduced energy spectrum of the π^0 in η decay. The experimental points, also reduced by phase space, are taken from Crawford *et al.* (Ref. 5). Curves I, II, III, and IV correspond, respectively, to the following sets of values of $2\omega_r$ and γ^2: (1) 397 MeV, 0.10; (2) 397 MeV, 0.25; (3) 397 MeV, 0.08; (4) 410 MeV, 0.10.

$2\omega_r = 2\sqrt{2}\mu$ (curve I). An increase of γ^2 beyond this value (curve II) is clearly disfavored by the data. A small decrease in γ^2 (e.g., $\gamma^2=0.08$) improves the fit, by bringing about a sharper drop near the high-energy end (curve III). The same effect can also be brought about by a slight increase in the resonance energy. Indeed, since the available phase-space energy is now much closer to the resonance energy, the spectrum is quite sensitive to variations in ω_r, and curve IV with $\gamma^2=0.1$ and $2\omega_r=410$ MeV seems also to give a satisfactory fit.

Finally, the branching ratios $R(\tau'/\tau)$ and $R(\eta'/\eta)$ are shown in Table I. A change in the resonance width has very little effect on $R(\tau'/\tau)$ whose value $\gamma^2=0.1$ agrees well with the experimental figure of 0.299 0.018.[13] The quantity $R(\eta'/\eta)$, on the other hand, is more sensitive to variations in the width and it appears that γ^2 somewhat less than 0.1 is favored by the observed value of 0.83 ± 0.32,[5] though the statistics of the latter leave enough scope for improvement.

To conclude this section, a resonance energy around $2\sqrt{2}\mu$ and a width of 87 MeV ($\gamma^2=0.1$) give about the best fit to the τ and τ' data. The η data are better fitted by a slightly lower value of the width ($\gamma^2\approx0.08$), corresponding to $\Gamma_r\approx78$ MeV). The model has thus a strong resemblance to the one proposed by Brown and Singer[4,20] through a formal di-pion propagator, corresponding to our relativistic Breit-Wigner formula. One difference, however, consists in our consideration of some energy dependence of the width function, viz., $\Gamma=2\gamma k$, to reproduce the correct threshold behavior in the corresponding effective-range formula for the $\pi-\pi$ phase shifts.[7] Of course our range of variation of the width *at* resonance ($\Gamma_r\approx$ 79–90 MeV) roughly agrees with theirs (75–100 MeV). However, since the available energy in the case of τ decay is much less than the resonance energy, the quantity $2\gamma k$ in this case is effectively smaller than $\Gamma_r=2\gamma r$ which is a fixed number in Brown and Singer's model. In this connection we would suggest that a more detailed consideration of the dependence of the width parameter on energy could even help to reduce the (small) gap between our predicted widths 87 and 78 MeV for the τ and η cases, respectively. For example, a function of the form

$$\Gamma = 2\gamma k(1+a^2k^2)^{-1},$$

instead of $\Gamma=2\gamma k$ as considered in Sec. 2, would effectively reduce γ^2 for the η case, because the larger phase-space available to it would lend a larger weight to the factor $(1+a^2k^2)$ compared with the τ case. Such "damping" of the width parameter with energy is qualitatively consistent with its behavior known in the literature, e.g., in the case of the N_{33}^* resonance.[21] However, a detailed consideration of such finer features must await more accurate data on these events.

TABLE I. The 3π branching ratios for K and η.

$2\omega_r$, γ^2	$R(\tau'/\tau)$	$R(\eta'/\eta)$
397 MeV, 0.10	0.305	1.16
397 MeV, 0.25	0.288	1.30
420 MeV, 0.10	0.292	1.23
397 MeV, 0.08	···	1.09

4. DISCUSSION

It appears that the resonance model proposed in *B* is in rather good agreement with the η and τ decay results in several respects. Essentially the same conclusion has been reached by Brown and Singer.[20] However, in view of our long association with the problem, we still feel intrigued by the question as to how this model must be reconciled with the earlier negative results obtained from dispersion theory.

One might crudely suggest that the success of a model like this could be ascribed to the dominance of the di-pion pole of $J=I=0$ in a dispersion formula. However the interference of the "background integral" with the di-pion pole might alter this conclusion especially if the interference is of the wrong sign. An answer to this question was sought by us sometime ago, after the rather poor response to the $I=0$ resonance model at the Rochester Conference of 1960.[8] For this purpose we used a formal Schrödinger equation for the final three-pion state,[22] taking the weak interaction to be structureless in momentum space. For the case of attractive $\pi-\pi$ interaction in the $I=0$ state, our result (unfortunately) confirmed the conclusion of Ref. 3 and therefore appeared in disharmony with the "resonance model" results of Refs. 7 and 8. On the other hand, an s-wave interaction which looked *repulsive* at low energies, seemed to produce a correct trend for the unlike pion spectrum. In terms of a "disperion formula" this can be interpreted to imply that only for a *repulsive* interaction

[20] L. Brown and P. Singer, Phys. Rev. **133**, B3812 (1964).

[21] See, e.g., M. Gell-Mann and K. M. Watson, Ann. Rev. Nucl. Sci. 4, 219 (1954).

[22] A. N. Mitra and S. Ray, Ann. Phys. (N. Y.) **21**, 439 (1963).

at *low* energies does the background integral interfere constructively, so to say, with the di-pion pole, provided of course that the existence of the resonance pole is not incompatible with a $\pi-\pi$ interaction which gives negative phase shifts at low energies. For attractive $\pi-\pi$ interaction, on the other hand, the interference is definitely of the wrong sign.[23] Actually very little is as yet known about the sign of the $\pi-\pi$ phase shifts at low energies. The theoretical S-wave phase shifts have passed through many vicissitudes since the early days of the Chew-Mandelstam[24] S-wave dominant solutions. For some time it was believed on the basis of the ABC anomaly,[25] $\pi-N$ scattering analysis,[26] etc., that the low-energy S-wave $\pi-\pi$ interaction was attractive (and stronger in $I=0$ than in $I=2$). However, the analysis of $\pi-N$ scattering does not entirely preclude negative phase shifts at least at low energies.[27]

Since the present results on τ and η are compatible with the earlier calculations[3,23] only on the basis of a "repulsive-looking" interaction, it may be interesting to see if such an interaction can actually produce an $I=J=0$ resonance at a sufficiently high energy. It is not difficult to visualize that certain special kinds of S-wave interaction (e.g., an attractive well surrounded by a repulsive wall), can, in principle, produce such an effect. Indeed, one of us has shown elsewhere[28] that for the particular (nonlocal) shape of $\pi-\pi$ interaction considered in Ref. 22, viz.,

$$(\mathbf{p}|V|\mathbf{q})=\lambda(\beta^2+p^2)^{-1}(\beta^2+q^2)^{-1}, \quad \lambda>0,$$

the $I=0$ $\pi-\pi$ phase shift, starts out with negative values at low energies, but becomes truly resonant at a sufficiently high energy, with a rather large width. As we have seen in Sec. 3, this is just the kind of behavior needed to produce the desired effects on the pion spectra in τ and η decays. In this connection, it has very recently come to our notice that Blankenbecler *et al.*[29] have arrived at very similar conclusions about the S-wave $\pi-\pi$ amplitude (repulsive at low energies, and resonant at a higher energy with a large width) from entirely different premises, namely, considerations of unitarity, crossing symmetry, and effect of several high mass channels. If such a scheme of $\pi-\pi$ interaction in $I=0$ is taken seriously, there is no disagreement in principle, between the present results and the earlier predictions on the pion spectra in τ decay.

It is important to note that this $\pi-\pi$ mechanism is different from the one which leads to an ABC anomaly. The latter is a manifestation of an inherently attractive low-energy interaction which can never produce an S-wave resonance. To guard against the "adverse effects" of an ABC type $\pi-\pi$ interaction in the present problem, all that is necessary is to postulate the production at source of a $\pi-\pi$ resonance of $I=0$ whose decay characteristics at low energies would automatically manifest as a repulsive $\pi-\pi$ interaction. As for the effects of any additional ABC type interaction (attractive) in the final three-pion state, the results of Ref. 22 would warrant the conclusion that such effects are not large.

As a final remark, the model proposed here offers only an alternative mechanism for the 3π modes of K and η decay. We do not have any reliable criteria to discriminate between the present model and other mechanisms, especially the successful pion-pole model,[2] except to suggest a more accurate analysis of the angular distribution of the unlike pion. Nor have we any comments to make on the direct experimental observability of such a $\pi-\pi$ resonance,[30] though we would like to point out that a width disproportionately large compared with the resonance energy would greatly complicate the possibilities of detection.

ACKNOWLEDGMENT

We are indebted to Professor R. C. Majumdar for his interest in this investigation.

[23] This last statement is not inconsistent with Kacser's (Ref. 1) observations which do not, however, cover the case of repulsive phase shifts at low energies.

[24] G. Chew and S. Mandelstam, Phys. Rev. **119**, 478 (1960).

[25] T. N. Truong, Phys. Rev. Letters **6**, 308 (1961).

[26] H. Hamilton, *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962).

[27] See, e.g., D. Atkinson, Phys. Rev. **128**, 1908 (1962).

[28] A. N. Mitra, Nuovo Cimento (to be published).

[29] R. Blankenbecler *et al.* (to be published).

[30] N. Samios, A. H. Bachman, R. M. Lea, T. E. Kalogeropoulos, and W. D. Shephard, Phys. Rev. Letters, **9**, 139 (1962). However, J. Steinberger, in Proceedings of the Siena Conference on Resonant Particles (to be published), has produced contrary evidence. One of us (ANM) is indebted to Professor M. G. K. Menon for bringing this last point to our notice.